س ۱) دوره‌ی بیستم

جواب سؤال ۱

الف)

$$t_0 < \frac{2v_0 \sin\theta}{g+a_0}$$

ب)

$\Delta t_1 = \Delta t_0$: شتاب $g+a_0$

Δt_2 : شتاب $g-a_0$

$\Delta t_{\text{پرواز}} = \Delta t_1 + \Delta t_2$

$$y(t_0) = -\frac{1}{2}(g+a_0)t_0^2 + v_0 \sin\theta t$$

$$\dot{y}(t_0) = v_0 \sin\theta - (g+a)t_0$$

معادله‌ی حرکت از t_0 تا رسیدن به زمین:

$$\Delta y = -y(t_0) = -\frac{1}{2}(g-a_0)\Delta t_2^2 + \dot{y}_{(t_0)}\Delta t_2$$

$$\Rightarrow \Delta t_2 = \frac{+\dot{y}_{(t_0)} \pm \sqrt{\dot{y}^2_{(t_0)} + 2(g-a_0)y_{(t_0)}}}{+(g-a_0)}$$

بدیهی است جواب منفی غیرقابل قبول است زیرا در این صورت Δt_2 منفی می‌شود.

$$R_{\text{برد}} = v_0 \cos\theta \,(t_0 + \Delta t_2)$$

ج)

حالت اول : $v_0 \sin\theta - (g+a_0)t_0 \leqslant 0 \rightsquigarrow T_{اوج} = \frac{v_0 \sin\theta}{g+a_0}$

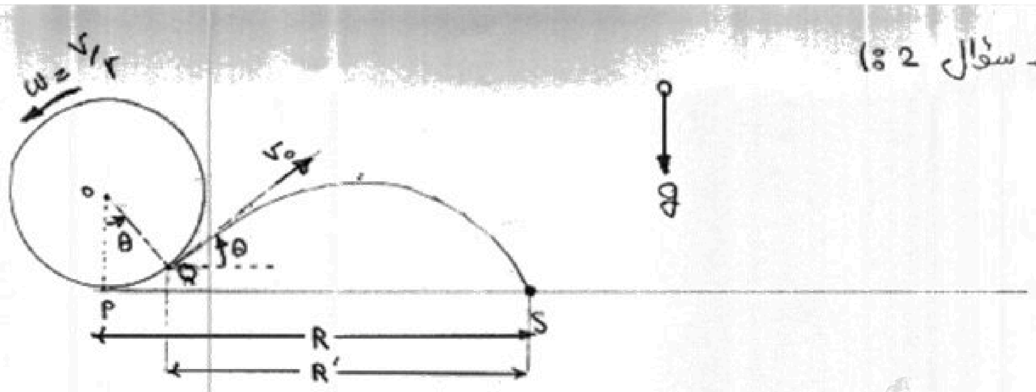
$$\to h_{اوج} = \frac{v_0^2 \sin^2\theta}{2(g+a_0)}$$

حالت دوم : $v_0 \sin\theta - (g+a_0)t_0 > 0$

$$h_{اوج} = h_{(t_0)} + \frac{\dot{y}^2_{(t_0)}}{2(g-a)}$$

$$\to h_{اوج} = -\frac{1}{2}(g+a_0)t_0^2 + v_0 \sin\theta\, t + \frac{(v_0 \sin\theta - (g+a_0)t_0)^2}{2(g-a_0)}$$

ـــ سؤال 2 :)



$$L = v^2/g \ , \ \alpha = rg/v^2 \implies r = \alpha L$$

الف)

$$R = r\sin\theta + R'$$

$$t_{\text{پرواز}} = v_0 \sin\theta/g + \sqrt{\frac{2h_{avg}}{g}} \ , \ h_{avg} = r(1-\cos\theta) + \frac{v_0^2 \sin^2\theta}{2g}$$

$$\Rightarrow R' = v_0\cos\theta \times t_{\text{پرواز}} = v_0 \cos\theta \times \left(\frac{v_0 \sin\theta}{g} + \sqrt{\frac{2}{g}\left(r(1-\cos\theta) + \frac{v_0^2 \sin^2\theta}{2g} \right)} \right)$$

$$\Longrightarrow R = \alpha L \sin\theta + R'$$

$$R' = L\sin\theta\cos\theta + \sqrt{\frac{2v_0^2\cos^2\theta}{g}\left(r(1-\cos\theta) + \frac{v_0^2\sin^2\theta}{2g} \right)}$$

$$= L\sin\theta\cos\theta + \sqrt{2L\cos^2\theta \left(\alpha L(1-\cos\theta) + \frac{L\sin^2\theta}{2} \right)}$$

$$\Longrightarrow R = \alpha L\sin\theta + L\sin\theta\cos\theta + \sqrt{2L\cos^2\theta\left(\alpha L(1-\cos\theta) + \frac{L\sin^2\theta}{2} \right)}$$

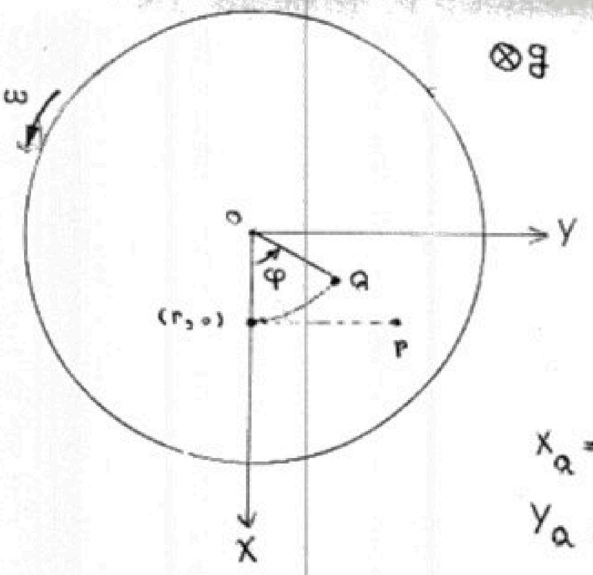
ب)

$$\left.\frac{dR}{d\theta}\right|_{\theta=\theta_0} = 0$$

$$\Rightarrow \alpha L\cos\theta_0 + L\cos 2\theta_0 + \frac{L}{2}\cos\theta_0 \frac{2\alpha\sin\theta_0 + 2\sin\theta_0\cos\theta_0}{\sqrt{2(\alpha(1-\cos\theta_0)) + \sin^2\theta_0}}$$

$$= L\sin\theta_0\sqrt{2\alpha(1-\cos\theta_0) + \sin^2\theta_0}$$

سه سؤال ۳ و)



الف) $t = \frac{2V}{g}$

ب) $x_P = r$

$Y_P = t \times \omega r$

$= 2\omega r V/g = r\theta$

ج) $X_Q = r\cos\varphi = r\cos\omega t = r\cos\theta$

$Y_Q = r\sin\varphi = r\sin\omega t = r\sin\theta$

د)

$$D = \sqrt{(X_P - X_Q)^2 + (Y_P - Y_Q)^2}$$

$$= \sqrt{r^2(1-\cos\theta)^2 + r^2(\theta - \sin\theta)^2} = r\sqrt{1 - 2\cos\theta + 1 + \theta^2 - 2\theta\sin\theta}$$

$$\Rightarrow D_{PQ} = r\sqrt{2 - 2\cos\theta - 2\theta\sin\theta + \theta^2}$$

ه)

$$\frac{\vec{OQ} \cdot \vec{PQ}}{|OQ||PQ|} = \cos\alpha$$

$$\vec{OQ} \cdot \vec{PQ} = r\cos\theta \times (r\cos\theta - r) + r\sin\theta \times (r\sin\theta - r\theta)$$

$$= r^2\left[\cos^2\theta - \cos\theta + \sin^2\theta - \theta\sin\theta\right] = r^2\left[1 - \cos\theta - \theta\sin\theta\right]$$

$$|OQ| = r \quad , \quad |PQ| = r\sqrt{2 - 2\cos\theta - 2\theta\sin\theta + \theta^2}$$

$$\Rightarrow \cos\alpha = \frac{1 - \cos\theta - \theta\sin\theta}{\sqrt{2 - 2\cos\theta - 2\theta\sin\theta + \theta^2}} \quad , \left(\frac{1}{\cos\alpha}\right)^2 - 1 = \tan^2\alpha$$

$$\Rightarrow \tan\alpha = \sqrt{\left(\frac{1}{\cos\alpha}\right)^2 - 1}$$

$$\tan^2\alpha = \frac{2-2\cos\theta-2\theta\sin\theta+\theta^2}{1+\cos^2\theta+\theta^2\sin^2\theta-2\theta\sin\theta-2\cos\theta+2\theta\sin\theta\cos\theta}-1$$

$$= \frac{2-\cancel{2\cos\theta}-\cancel{2\theta\sin\theta}+\theta^2-1-\cos^2\theta-\theta^2\sin^2\theta+\cancel{2\theta\sin\theta}+\cancel{2\cos\theta}-\theta\sin 2\theta}{1+\cos^2\theta+\theta^2\sin^2\theta-2\theta\sin\theta-2\cos\theta+2\theta\sin\theta\cos\theta}$$

$$= \frac{1-\cos^2\theta+\theta^2(1-\sin^2\theta)-\theta\sin 2\theta}{1+\cos^2\theta+\theta^2\sin^2\theta-2\theta\sin\theta-2\cos\theta+\theta\sin 2\theta} = \frac{1-\cos^2\theta-\theta\sin 2\theta+\theta^2(1-\sin^2\theta)}{(1-\cos\theta-\theta\sin\theta)^2}$$

$$\Rightarrow \tan\alpha = \frac{\sqrt{1-\cos^2\theta-\theta\sin 2\theta+\theta^2(1-\sin^2\theta)}}{1-\cos\theta-\theta\sin\theta}$$

سؤال ۴ سه

الف)

$$x > f(h): \quad \frac{x - f(h)}{f(h)} \rightsquigarrow y = h\left(\frac{x}{A+Bh} - 1\right)$$

$$x < f(h): \quad y = h\left(1 - \frac{x}{A+Bh}\right)$$

ب)

$$(1) \quad \frac{dy}{dh} = 0 \rightsquigarrow h = \frac{\sqrt{xA} - A}{B}$$

$$\rightsquigarrow Y_1 = \frac{A + x - 2\sqrt{xA}}{B}$$

$$(2) \quad Y_2 : h \to D \Rightarrow Y_2 = D\left(1 - \frac{x}{A+BD}\right)$$

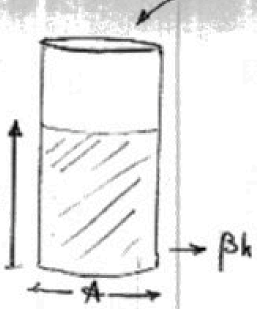
ج) Y_2 نزولی و Y_1 صعودی است ماکسیمم Yها تا نقطه تقاطعشان است و مینیمم هست

در $Y_1 = Y_2$ است

$$\rightsquigarrow x = \frac{16}{9} BD$$

$$\Longrightarrow R/D = \frac{1}{9}$$

سؤال 5 :



$\alpha(H-h) \rightarrow 0 \rightarrow T_1$

$T_1+T_2 \rightarrow 2T_1+T_2$

$2(T_1+T_2) \rightarrow 3T_1+2T_2$

$\vdots$

$$h_{(n(T_1+T_2))} = u_n$$

$$h\,(n(T_1+T_2)+T_1) = v_n$$

الف)

$$V = Ah \Rightarrow \dot{V} = A\dot{h}$$

$$\dot{h}_{\text{باز}} = \frac{1}{A}\left(\alpha(H-h) - \beta h\right)$$

ب)

$$\dot{h}_{\text{بسته}} = \frac{-1}{A}(\beta h) = -\frac{\beta h}{A}$$

ج)

$$\dot{h}_{\text{باز-اول}} = \frac{1}{A}\left(\alpha(H-u_n) - \beta u_n\right)$$

$$\dot{h}_{\text{باز-آخر}} = \frac{1}{A}\left(\alpha(H-v_n) - \beta v_n\right)$$

$$\Rightarrow \dot{h} \simeq \frac{1}{A}\left[\alpha H - (\alpha+\beta)\frac{u_n+v_n}{2}\right]$$

$$\Rightarrow \alpha(H) - (\alpha+\beta)h = \alpha(H) - (\alpha+\beta)(u_n+v_n)/_2$$

$$\Rightarrow h = \frac{u_n+v_n}{2}$$

$$\Rightarrow \frac{dh}{dt} = \cancel{\quad} \;\dot{h} \Rightarrow v_n - u_n = \frac{T_1}{A}\left[\alpha H - (\alpha+\beta)\frac{u_n+v_n}{2}\right]$$

د)

$$\dot{h}_{\text{نقطه اول}} = -\frac{\beta}{A}v_n$$

$$\Rightarrow \quad \dot{h} = -\frac{\beta}{A}\left(\frac{v_n + u_{n+1}}{2}\right)$$

$$\dot{h}_{\text{نقطه آخر}} = -\frac{\beta}{A}u_{(n+1)}$$

$$h = \frac{v_n + u_{n+1}}{2}$$

$$u_{n+1} - v_n = -\frac{\beta T_2}{A}\left(\frac{v_n + u_{n+1}}{2}\right)$$

ه)

$$u_{n+1} = u_n$$

$$\Rightarrow \quad \frac{T_1}{A}\left[\alpha H - (\alpha+\beta)\frac{u_n + v_n}{2}\right] = \frac{\beta T_2}{A}\left(\frac{v_n + u_n}{2}\right)$$

$$\Rightarrow 2\alpha H = \left[(\alpha+\beta) + \frac{\beta T_2}{T_1}\right](v_n + u_n)$$

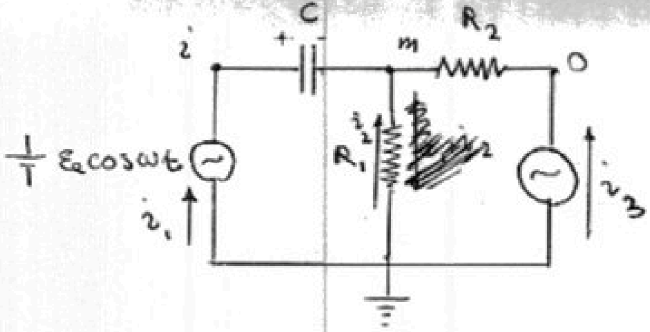
$$\Rightarrow u_n + v_n = \frac{2\alpha H}{\alpha + \beta + \frac{\beta T_2}{T_1}}$$

$$u_n - v_n = -\frac{\beta T_2}{A}\left(\frac{\alpha H}{\alpha + \beta + \beta T_2/T_1}\right)$$

$$\Rightarrow u_n = \frac{\alpha H}{\alpha + \beta + \beta T_2/T_1} - \frac{\beta T_2}{2A}\left(\frac{\alpha H}{\alpha + \beta + \beta T_2/T_1}\right)$$

$$v_n = \frac{\alpha H}{\alpha + \beta + \beta T_2/T_1} + \frac{\beta T_2}{2A}\left(\frac{\alpha H}{\alpha + \beta + \beta T_2/T_1}\right)$$

مسئله سؤال 6:



الف)

$$\varepsilon\cos\omega t - \frac{q}{C} = V_m \implies -\varepsilon\omega\sin\omega t - \frac{\dot q}{C} = \dot V_m$$

$$-i_2R_1 = V_m \implies i_2 = \frac{-V_m}{R_1}$$

$$AV_m - i_3R_2 = V_m \implies i_3 = V_m(A-1)/R_2$$

$$-\dot q = i_2 + i_3$$

$$\implies \dot V_m + \varepsilon\omega\sin\omega t = \frac{V_m}{C}\left(-\frac{1}{R_1} + \frac{(A-1)}{R_2}\right)$$

$$V_m = K_1\cos\omega t + K_2\sin\omega t$$

$$\dot V_m = -K_1\omega\sin\omega t + K_2\omega\cos\omega t$$

$$-K_1\omega\sin\omega t + K_2\omega\cos\omega t + \varepsilon\omega\sin\omega t = K_0(K_1\cos\omega t + K_2\sin\omega t)$$

$$-K_1\omega + \varepsilon\omega = K_0K_2 \qquad \varepsilon\omega = K_1\left(\frac{K_0^2}{\omega} + \omega\right) \implies K_1 = \frac{\varepsilon\omega^2}{\omega^2 + K_0^2}$$

$$K_2\omega = K_0K_1$$

$$K_2 = \frac{\varepsilon\omega^2 K_0/\omega}{\omega^2 + K_0^2}$$

$$\implies a = \frac{A\varepsilon\omega^2}{\omega^2 + K_0^2}$$

$$K_0 = -\frac{1}{C}\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{A}{R_2}\right)$$

$$b = \frac{A\varepsilon\omega k_0}{\omega^2 + K_0^2}$$

ب)

$$A \to \infty \begin{cases} a \to 0 \\ b \to \dfrac{\mathcal{E}_0 \omega}{\frac{1}{C} \cdot \frac{1}{R_2}} = \mathcal{E}_0 \omega R_2 C \end{cases} \Rightarrow V_0 = \mathcal{E}_0 \omega R_2 C \sin \omega t$$

جواب سؤال ۷:

الف)

$$v_n = \sqrt{v_0^2 + 2nqV/m}$$

ب)

$$L_n = v_n T$$

$$\Rightarrow L_n = T\sqrt{v_0^2 + 2nqV/m}$$

ج)

$$t = \frac{T\sqrt{v_0^2 + 2nqV/m}}{\sqrt{v_0^2 + 2nqV/m + \epsilon}} = T\left(1 - \frac{\epsilon/2}{v_0^2 + 2nqV/m}\right)$$

د)

$$\Delta t = \sum_1 T\left(\frac{\epsilon}{2} \cdot \frac{1}{v_0^2 + 2nqV/m}\right)$$

$$= \frac{T\epsilon}{2} \sum_1 \left(\frac{m/2qV}{n + \frac{mv^2}{2qV}}\right) = \frac{mT\epsilon}{4qV} f\left(K, \frac{mv^2}{2qV}\right)$$

جواب سؤال ۸:

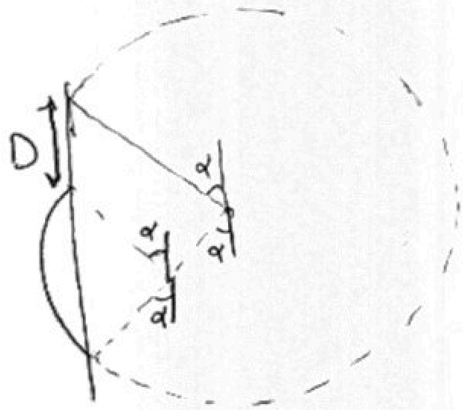
الف)

$$r_1 = \frac{mV}{qB_1}$$

$$\omega t_1 = \pi - 2\alpha$$

$$\Rightarrow \frac{qB_1}{m} t_1 = \pi - 2\alpha$$

$$\Rightarrow t_1 = \frac{(\pi - 2\alpha) m}{qB_1}$$



ب)

$$r_2 = \frac{mV}{qB_2} \quad , \quad t_2 = \frac{(\pi - 2\alpha) m}{qB_2}$$

ج)

$$2 r_2 \cos\alpha - 2 r_1 \cos\alpha = D$$

$$\Rightarrow D = 2\cos\alpha \frac{mV}{q} \left(\frac{1}{B_2} - \frac{1}{B_1} \right)$$

د)

$$\frac{D}{t_2} = \frac{2\cos\alpha \frac{mV}{q} \left(\frac{1}{B_2} - \frac{1}{B_1} \right)}{(\pi - 2\alpha) m / qB_2} = \frac{2\cos\alpha}{\pi - 2\alpha} V \left(1 - \frac{B_2}{B_1} \right)$$

جواب سؤال ۹ سه

الف)

$$v\cos\alpha\, t > l$$

$$\frac{1}{2}at^2 = d + l\tan\alpha$$

$$\Rightarrow \frac{1}{2}a\frac{l^2}{v^2\cos^2\alpha} = d + L\tan\alpha$$

$$\Rightarrow v^2 = \frac{aL^2}{2}.\sec^2\alpha.\frac{1}{d+L\tan\alpha}$$

$$\Rightarrow v = L\sqrt{\frac{a/2}{\cos^2\alpha(d+L\tan\alpha)}}$$

ب)

$$\frac{dv}{d\alpha} = 0 \Rightarrow d\left[\cos^2\alpha(d+L\tan\alpha)\right]/d\alpha = 0$$

$$\Rightarrow -2\sin\alpha\cos\alpha(d+L\tan\alpha) + \cos^2\alpha(L\sec^2\alpha) = 0$$

$$\Rightarrow L = 2\sin\alpha\cos\alpha(d+L\tan\alpha) \rightarrow 1 = 2d/L\sin\alpha\cos\alpha + 2\sin^2\alpha$$

$$\Rightarrow 1-2\sin^2\alpha = \cos 2\alpha = d/L\sin 2\alpha \rightarrow \tan 2\alpha = \frac{L}{d}$$

$$\Rightarrow \alpha = \frac{1}{2}\tan^{-1}\left(\frac{L}{d}\right)$$

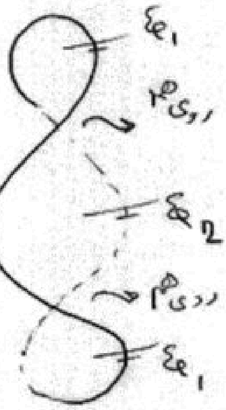
ج) در ب "جاگذاری می‌کنیم و ساده می‌کنیم

سؤال 15 سه

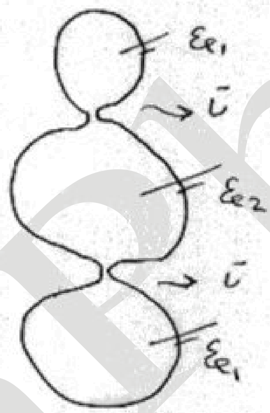
سه حالت ممکن است :

$$\varepsilon_1 = \pi r_1^2 \frac{\Delta B}{\Delta t}$$

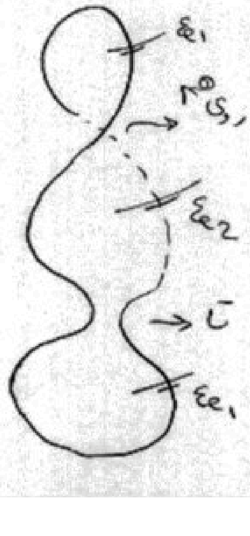
$$\varepsilon_2 = \pi r_2^2 \frac{\Delta B}{\Delta t}$$



$$\Rightarrow i = \frac{1}{R}\left(\pi r_2^2 - 2\pi r_1^2\right)\frac{\Delta B}{\Delta t}$$



$$i = \frac{1}{R}\left(\pi r_2^2 + 2\pi r_1^2\right)\frac{\Delta B}{\Delta t}$$



$$i = \frac{1}{R}\left(\pi r_2^2 - \pi r_1^2 + \pi r_1^2\right)\frac{\Delta B}{\Delta t}$$

$$= \frac{1}{R}\left(\pi r_2^2\right)\frac{\Delta B}{\Delta t}$$